Luminosity *vs.* Line-Width for Spiral Galaxies

Traditionally, the most accepted measurement of the Hubble Constant comes from the luminosity-line width relation for spiral galaxies, otherwise known as the Tully-Fisher relation. Since the technique uses star-forming galaxies, it can be well-calibrated with Cepheids, and, since it involves the luminosity of entire galaxies, it can easily be used to measure the undisturbed Hubble Flow. Unfortunately, the technique has arguably produced more controversy than any other relation in astronony.

The idea that that rotation rate of a spiral galaxy might be useful as a distance indicator goes back to Öpik in 1922, but the first modern formulation was presented by Mort Roberts in the 1960's. Consider a system where some H I is distributed significantly outside the galaxy's main mass concentration. According to Kepler's law, the period of rotation of this H I gas is related to galactic mass by

$$\mathcal{M}_{\rm tot} = \frac{r^3}{P^2} \tag{15.01}$$

where we assume a circular orbit and r is the distance of the gas from the galactic center. The rotational velocity of this gas is

$$v_c = \frac{2\pi r}{P} \tag{15.02}$$

so

$$\mathcal{M}_{\rm tot} = r^3 \left(\frac{v_c}{2\pi r}\right)^2 = \frac{r v_c^2}{4\pi^2} \tag{15.03}$$

or, since the radius is related to the observed angular size by the distance $r = d\theta$

$$\mathcal{M}_{\rm tot} \propto \theta\, d\, v_c^2 \tag{15.04}$$

Now suppose we group galaxies together such that, for a class of galaxies, the ratio of hydrogen gas to total galactic mass is a

constant. In other words, $\langle g \rangle = \mathcal{M}_{\rm H\ I}/\mathcal{M}_{\rm tot}$. Thus

$$\mathcal{M}_{\rm H\ I} \propto \theta\, d\, v_c^2 \tag{15.05}$$

Since the mass of H I present in a galaxy is related to the total flux emitted at 21 cm

$$\mathcal{M}_{\rm H\ I} \propto d^2 f_{21} \propto \theta\, d\, v^2 \tag{15.06}$$

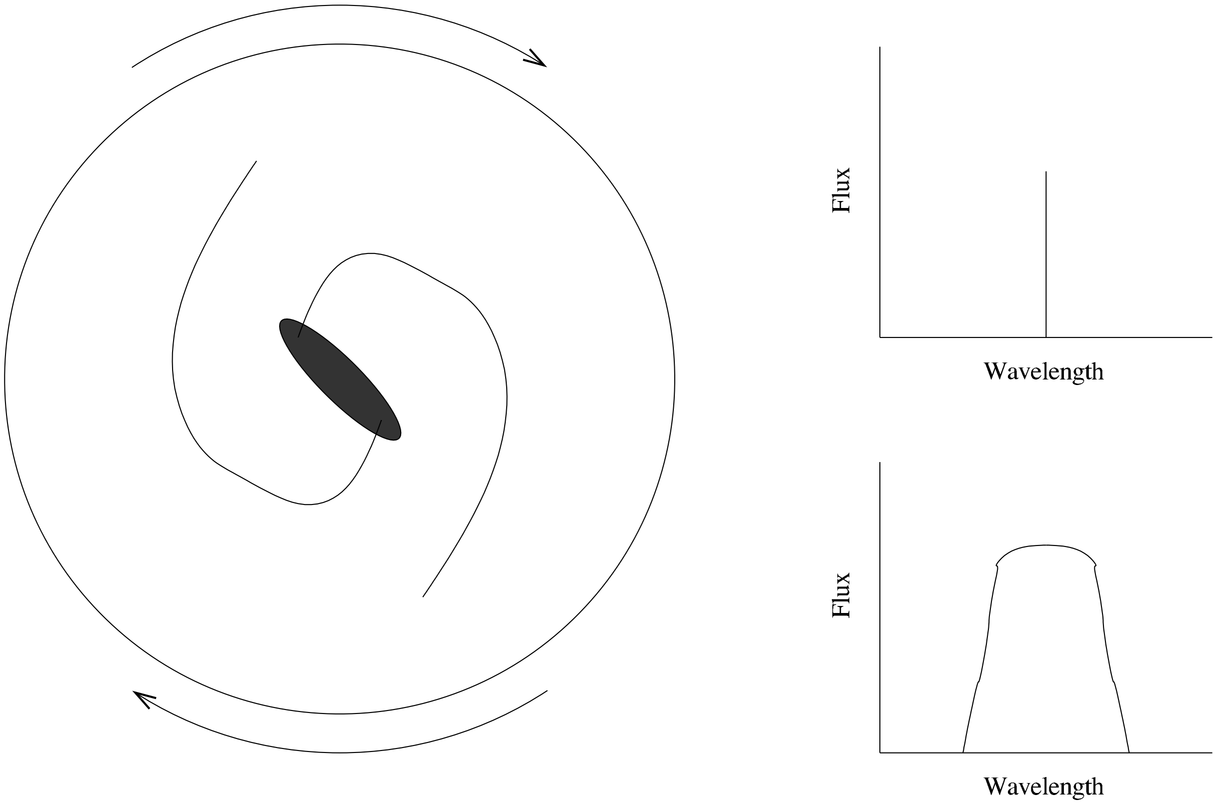
where f_{21} is the observed 21 centimeter flux. Consequently,

$$d \propto \frac{\theta\, v_c^2}{f_{21}} \tag{15.07}$$

Once could therefore observe nearby galaxies to determine the constant of proportionality, and then use the relation between total 21 centimeter flux and rotation velocity to derive galactic distances.

Note here that radio telescopes like Arecibo do not have high spatial resolution. Therefore, unless the galaxy is nearby (*i.e.*, has a *large* angular size) or is observed with a large array, it is difficult to determine the motion of the gas at particular locations in the galaxy. However, you can measure the total Doppler width of the H I emission, and the width of the emission line reflects the galaxy's rotation. This is the way large samples of galaxies are investigated.

(Roberts' formulation for a luminosity-line-width relation was never seriously investigated, though it did prove to be a motivating factor for today's Tully-Fisher Relation.)



In most radio telescope observations, galaxies are unresolved (or just marginally resolved).

The Tully-Fisher Relation

Perhaps the most important relation in extragalactic astronomy is the luminosity-line width relation for spiral galaxies, otherwise known as the Tully-Fisher relation. Its existence not only has implications for the extragalactic distance scale, but also for the structure of galaxies, the distribution of dark matter, the formation of galaxies, and large-scale structure in the universe.

Once again, let's start with the idea that the rotation rate of a galaxy is somehow related to its mass. From above

$$\frac{\mathcal{M}_{\rm tot}}{r} \propto v_c^2 \tag{15.08}$$

Now, let's generalize this equation by saying that the quantity $\mathcal{M}/r$ is proportional to galactic luminosity, and that the coefficient of v_c need not be 2. Thus

$$\mathcal{L} \propto v_c^\alpha \tag{15.09}$$

or, in terms of magnitudes

$$M = a \log v_c + b \tag{15.10}$$

Or, if we note explicitly that v_c is not observed, but the Doppler line width (W) is, then

$$M = a \log W + b \tag{15.11}$$

This is the modern form of the Tully-Fisher relation. Once calibrated (via nearby galaxies with known distances), a galaxy's observed line-width can be used to determine its absolute magnitude. A comparison with apparent magnitude then yields the distance modulus.

Although simple in theory, the details of the observations can be tricky. First, the observed rotation rate (or line width) of a galaxy depends on the galaxy's orientation to the line-of-sight. Specifically, the observed rotation rate is related to the true rotation rate via the galaxy's inclination,

$$v_{true} = v_{obs} / \sin i \tag{15.12}$$

so an estimate of inclination is a must. (Note that for highly inclined galaxies, $\sin i \approx 1$, so that the correction to line-width is small for edge-on systems.) In addition, even in the case of a perfectly face-on system, the emission-line widths have finite width, due to the z-motions in a galaxy. So one needs to know how important this component of "turbulence" is.

In addition, galactic internal extinction will affect the measurement of the total apparent magnitude. The effect is largest at short wavelengths (say, B) and least in the infrared. But it is also inclination dependent. Highly inclined galaxies will have a larger fraction of their light extincted, since more of the galaxy's luminosity must past through the dust on its way to the observer.

The First Tully-Fisher Distances

[Fisher & Tully 1977, *Comm. Astrophys.*, **7**, 85]
[Tully & Fisher 1977, *Astr. Ap.*, **54**, 661]
[Sandage & Tammann 1976, *Ap. J.*, **210**, 7]

The first paper detailing the Tully-Fisher relation appeared in 1977; it concluded that the distance modulus to the Virgo Cluster was $(m-M) = 30.8$ (14.4 Mpc). Sandage and Tammann quickly published a rebuttal paper, which appeared in 1976: using the same data, and applying the same technique, they concluded that Virgo's distance modulus was $(m-M) = 31.45$ (19.5 Mpc). Why the difference?

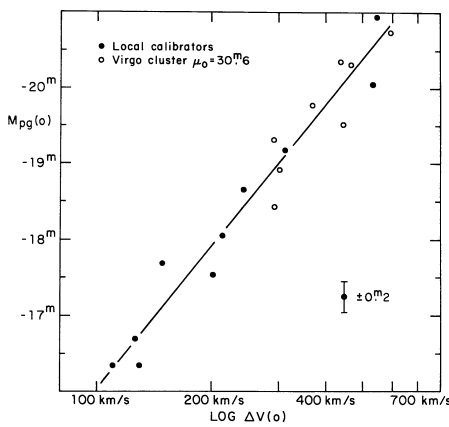
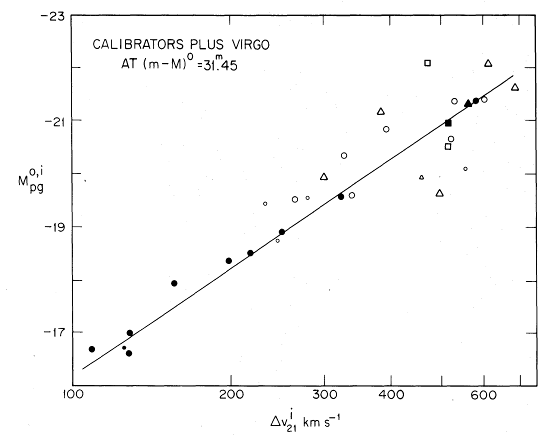


Fig. 5 (a) Absolute magnitude – global profile width relation produced by overlaying Figure 3 on Figure 1, adjusting Figure 3 vertically to arrive at a best visual fit with a distance modulus of $\mu_0 = 30^m.6 \pm 0^m.2$



Determining Inclination

To first order, one can estimate the inclination of a spiral galaxy by assuming that the galaxy is a infinitely thin disk. When viewed face-on, the observed axis ratio, $b/a = 1$; when viewed edge-on, $b/a = 0$, so under this approximation,

$$\cos i = b/a \tag{15.13}$$

Unfortunately, galaxies are not infinitely thin, so (15.13) breaks down at high inclinations. A better estimate is obtained by modeling a spiral galaxy as an oblate spheroid, with axis ratios a:a:c, where $c < a$. It's a relatively simply geometry problem to derive the formula

$$\cos^2 i = \frac{(b/a)^2 - \alpha^2}{1 - \alpha^2} \tag{15.14}$$

where $\alpha = c/a$ is the intrinsic axis ratio, and b/a is the observed ratio. For Sc galaxies, α is thought to be ~ 0.13.

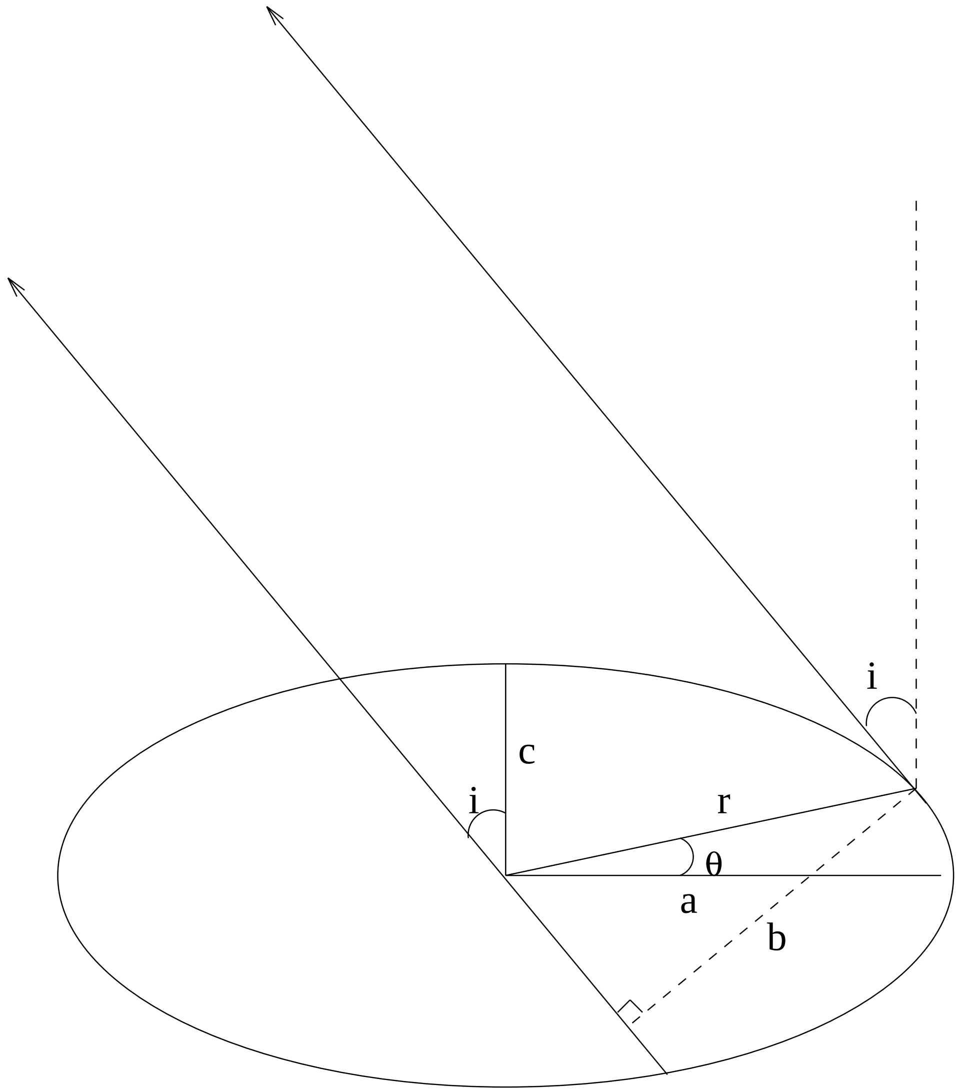
The derivation of the above equation is left as an exercise to the interested student, but to get you started, consider that the equation of an ellipse can be written in parametric form as

$$x = a\cos\theta \qquad\qquad y = c\sin\theta \tag{15.15}$$

To relate the size of the true minor axis to the size of the observed minor axis, consider that the latter is defined by the location along the ellipse where the slope is tangent to the line-of-sight. In other words, you need to know the value of θ where

$$\frac{dy}{dx} = \frac{dy}{d\theta}\Big/\frac{dx}{d\theta} = -\frac{c\cos\theta}{a\sin\theta} = -\frac{c}{a}\cot\theta = -\frac{c}{a}\tan(90+i) \tag{15.16}$$

The length of the observed minor axis is then the vector from the center of the ellipse to the tangent point, projected onto the



plane of the sky (which is defined as being perpendicular to i)

$$b = (a\cos\theta, c\sin\theta) \cdot (1, \frac{a}{c}\tan\theta) \tag{15.17}$$

After some algebra, the result is equation (15.14).

The above analysis, of course, requires that the observer know the true axis ratio, α. This can be estimated by observing a large number of galaxies, and seeing how the observed distribution of b/a agrees with the prediction. For an ensemble of randomly oriented galaxies, the inclinations should be distributed as $\sin i\, di$. So

$$\cos^2 i = \frac{(b/a)^2 - \alpha^2}{1 - \alpha^2} \tag{15.18}$$

implies

$$\sin i\, di = (b/a) \left\{ \left[(b/a)^2 - \alpha^2 \right] \left[1 - \alpha^2 \right] \right\}^{-1/2} d(b/a) \tag{15.19}$$

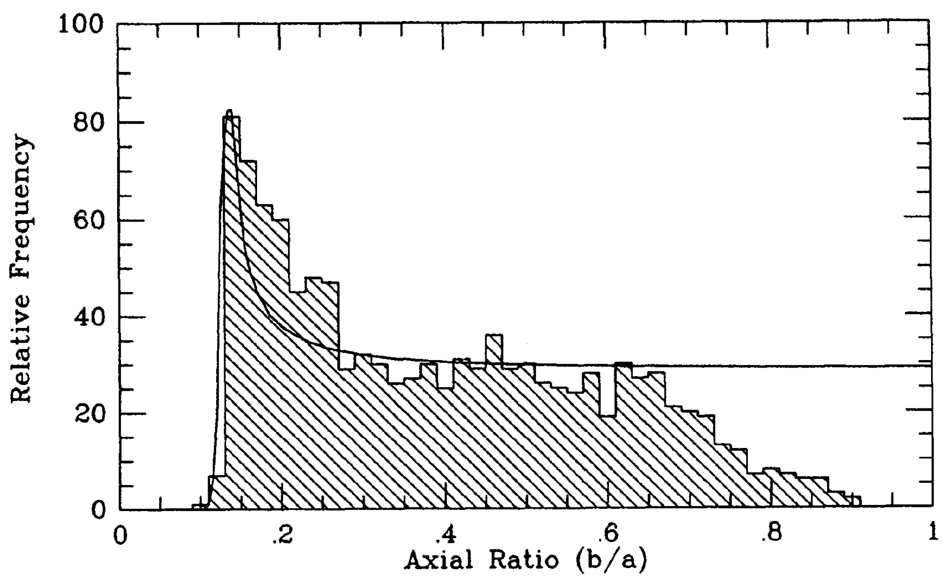


FIG. 7. Histogram of corrected minor to major diameter ratios for the galaxies in our sample. The dashed line is the distribution function given by Eq. (19) with $q=0.13$. Curve and histogram are arbitrarily scaled so that the areas under them for $b/a<0.7$ are the same.

[Giovanelli *et al.* 1994, *A.J.*, **107**, 2036]

Inclination Errors

Inclination measurements are critical for the success of a Tully-Fisher analysis, and any error in i will propagate directly into the derived distance. Consider a galaxy with observed axis ratio b/a. This ratio, naturally, will have a small amount of measurement error, $\sigma_{b/a}$. Let's see how this error propagates into the estimation of the rotational velocity. For simplicity, we'll assume spiral galaxies are infinitely thin disks (the mathematics and result for the oblate spheroid is extremely similar). If $\epsilon = b/a$, then $\cos i = \epsilon$, and from (15.12), the true rotational velocity is

$$v_t = v_{obs}/\sin i \tag{15.20}$$

so

$$v_t = \frac{v_{obs}}{(1-\cos^2 i)^{1/2}} = \frac{v_{obs}}{(1-\epsilon^2)^{1/2}} \tag{15.21}$$

According to the rules of error propagation,

$$\sigma_{v_t}^2 = \sigma_{v_{obs}}^2 \left(\frac{\partial v_t}{\partial \epsilon}\right)^2 \tag{15.22}$$

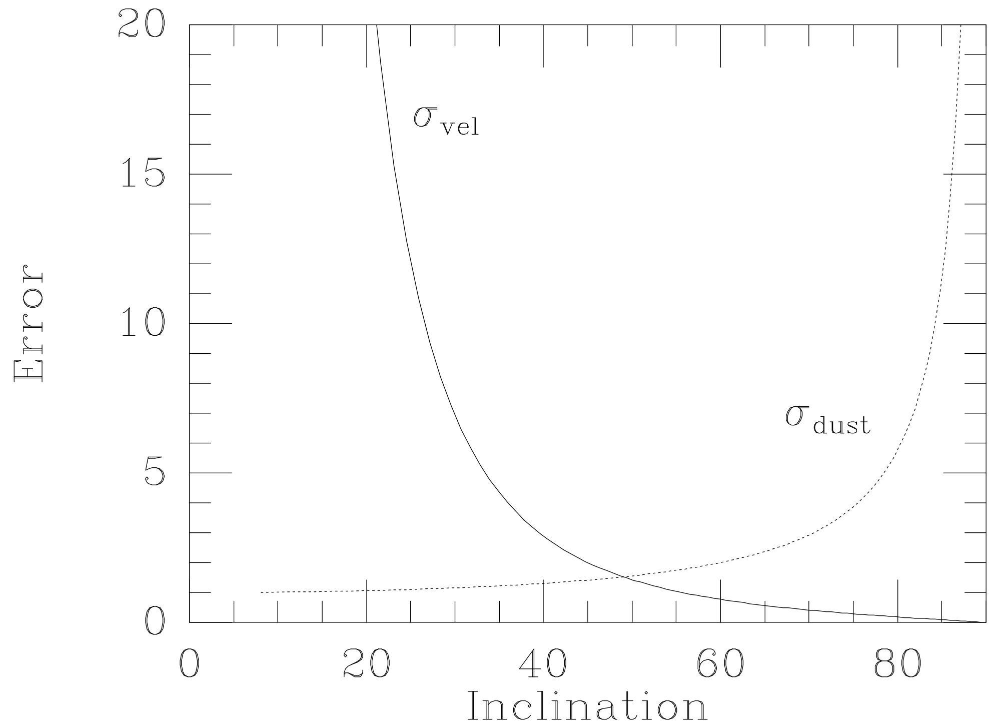
Doing the math yields

$$\sigma_{v_t} = \sigma_\epsilon v_{obs} \left\{ \frac{\epsilon}{(1-\epsilon^2)^{3/2}} \right\} \tag{15.23}$$

This is similar to the result one obtains for a non-zero value of α,

$$\sigma_{v_t} = \sigma_\epsilon v_{obs} (1-\alpha^2)^{1/2} \left\{ \frac{\epsilon}{(1-\epsilon^2)^{3/2}} \right\} \tag{15.24}$$

The interesting property about this function is that the term in brackets is extremely dependent on inclination: at high inclination, errors in inclination make very little difference, but at low inclination, a small uncertainty in b/a means a large error in the derived rotational velocity.



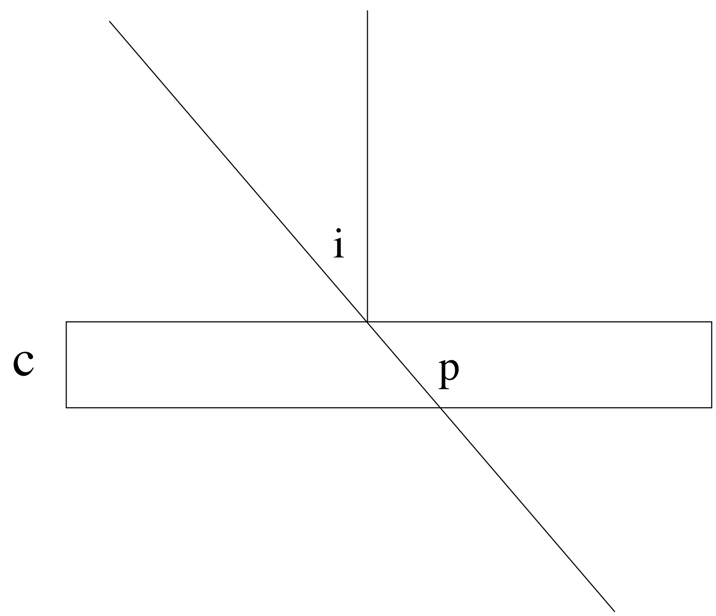
Errors in inclination will also affect the correction for galaxy luminosity (due to extinction). However, this uncertainty does not become terrible until very high $i > 80°$ inclinations.

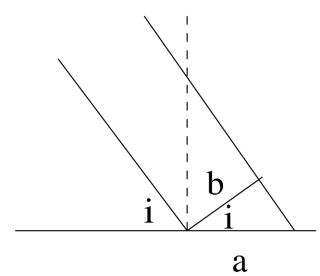
Tully-Fisher and Internal Extinction

[Holmberg 1958, *Medd. Lunds Obs. Ser. 2,* No. 136]
[Tully & Fouque 1985, *Ap.J. Supp.,* **58,** 67]

Dust can have an affect on the derived luminosities of galaxies. The zeroth order correction to this can be computed by assuming the dust layer is a simple slab parallel to the disk of the galaxy. In this case, the extinction is proportional to the path length through the dust, which, for an infinitely thin slab is

$$A_i = q \sec i \implies A_i - A_{i=0} = q \sec i - q \sec(0) = q(\sec i - 1) \tag{15.25}$$





Fits to Sc galaxies suggest, $q \sim 0.28$ mag (in the B-band). A better approximation is to say that the dust lies in a layer that cuts through the center of the galaxy. In other words, half the galaxy is extincted, and the other half (above the dust) is not, *i.e.*,

$$\mathcal{L} = 0.5\mathcal{L} + 0.5\mathcal{L}e^{-\kappa \sec i} \tag{15.26}$$

so

$$A_i = -2.5 \log \left\{0.5(1 + e^{-\kappa \sec i})\right\} \tag{15.27}$$

Or, one can do even better: one can assume the dust sheet has a finite thickness. In this case, a fraction of the light f, is in front of the dust, a fraction f is behind the dust, and the remainder is intermingled. A decent fit is obtained using

$$A_i = -2.5 \log \left\{ f(1 + e^{-\kappa \sec i}) + (1 - 2f) \left(\frac{1 - e^{-\kappa \sec i}}{\kappa \sec i} \right) \right\} \tag{15.28}$$

with $\kappa \sim 0.55$, $f \sim 0.25$, and i is never given a value greater than $80°$.

Alternatively, one can take an completely empirical approach. One can observe *large* samples of galaxies, divide them into redshift bins, and look for magnitude trends with size, surface brightness, and/or velocity line-width. Then you can just correct the data using a value ΔM that depends on inclination (and bandpass).

Using Tully-Fisher

In practice, the calibration of the Tully-Fisher is fairly straightforward.

• Choose a cluster of galaxies that are presumably all at the same distance

• Observe a complete sample of these objects at optical wavelengths (to determine their relative magnitudes), and at radio wavelengths (to measure their H I Doppler line-width). In general, to minimize the effects of internal extinction and variations in the present star-formation rate, it is best to do the optical observations in the red or infrared. (That way, a few bright blue O and B stars won't affect the galaxy's total luminosity that much.) Presently, CCD I-band observations are most common, but H-band aperture photometry has also been done extensively.)

• Plot magnitude versus the log of the line-width, and determine the slope of the Tully-Fisher relation.

• Measure the magnitude and Doppler line-widths for a few galaxies with known distances (and therefore known absolute magnitudes). Use these absolute magnitudes to determine the zero-point (intercept) of Tully-Fisher relation.

The *Hubble Space Telescope* was named that since one of its principle projects at inception was to measure the Hubble Constant to 10%. This task fell to the *HST* Distance Scale Key Project. Their measurement method was to determine the Cepheid distances to a dozen or so local, high-inclination spiral galaxies and calibrate the zero-point of Tully-Fisher.

Malmquist Bias

[Malquist 1922, *Lunn Medd. Ser.* **I,** 100, 1]

An extremely important issue in all astronomical observations is that of biases. Even the most complete survey is subject to biases of one type or another that can affect the result. One of the most famous (or infamous) of these biases is Malmquist bias. This bias muddled the issue of Tully-Fisher distances for almost three decades.

To understand what Malmquist bias is, let's consider the original problem that Malmquist set out to investigate – the luminosity of F-stars in the Galaxy. Suppose we are dealing with a *magnitude limited* sample of stars, *i.e.,* all objects brighter than a certain apparent magnitude are being analyzed. Let's also suppose that not all F-stars are exactly alike, but instead that there is a Gaussian distribution of absolute magnitudes, with mean magnitude M_0 and dispersion, σ. In other words, the true luminosity function of stars is

$$\Phi(M) = \Phi_0 e^{-(M-M_0)^2/2\sigma^2} \tag{16.01}$$

Now let $\rho(r)$ be the space density of stars, and $A(m)$ be the observed luminosity function, *i.e.,*

$$A(m) = \int_0^\infty \Phi(M)\rho(r)\, r^2 dr \tag{16.02}$$

or

$$A(m) = \int_0^\infty \Phi_0 e^{-(M-M_0)^2/2\sigma^2} \rho(r)\, r^2 dr \tag{16.03}$$

Now for our analysis, we will need the derivative of this function with respect to apparent magnitude. Since, from the definition of distance modulus,

$$\frac{d}{dM} = \frac{d}{dm} \tag{16.04}$$

we can write

$$\begin{aligned}
\frac{dA(m)}{dm} &= \int_0^\infty \Phi_0 \, \frac{d}{dM} \left\{ e^{-(M-M_0)^2/2\sigma^2} \right\} \rho(r)\, r^2 dr \\
&= \int_0^\infty \Phi_0 \left\{ -\frac{2(M-M_0)}{2\sigma^2} \right\} e^{-(M-M_0)^2/2\sigma^2} \rho(r)\, r^2 dr \\
&= \int_0^\infty \frac{M_0}{\sigma^2} \Phi(M) \rho(r)\, r^2 dr - \int_0^\infty \frac{M}{\sigma^2} \Phi(M) \rho(r)\, r^2 dr \\
&= \frac{M_0}{\sigma^2} A(m) - \frac{1}{\sigma^2} \int_0^\infty M \Phi(M) \rho(r)\, r^2 dr \qquad (16.05)
\end{aligned}$$

Now, if we divide by $A(m)$, we get

$$\frac{dA(m)}{dm} \Bigg/ A(m) = \frac{M_0}{\sigma^2} - \frac{1}{\sigma^2} \frac{\int_0^\infty M \Phi(M) \rho(r)\, r^2 dr}{\int_0^\infty \Phi(M) \rho(r)\, r^2 dr} \qquad (16.06)$$

But note that the last term is, by definition, the mean absolute magnitude of the sample. Thus

$$M_0 - \langle M \rangle = \sigma^2 \, \frac{d \ln A(m)}{dm} \qquad (16.07)$$

In other words, the mean absolute magnitude of the stars in the sample does not equal the intrinsic mean for the set of stars. In particular, the difference is proportional to the dispersion of the sample, squared. (If the intrinsic dispersion in the sample is large, you can be off by quite a bit!)

Malmquist bias comes about because of the artificial cutoff imposed by a magnitude limited sample. If the apparent magnitude cutoff is m, then more-or-less all objects out to a distance

$$d = 10^{0.2(m-M)+1} \tag{16.08}$$

are being observed. However, some exceptionally bright objects at larger distances will make it into the sample, while some exceptionally fainter objects that are closer than d will not. Thus, you are biased towards brighter objects.

Note that this is not just a minor inconvenience. Suppose you wanted to derive the distance to a galaxy by observing its F-stars. (They'd be too faint, but never mind that.) If you did not include the effects of Malmquist bias, then you would overestimate the true absolute magnitudes of F-stars in the Milky Way, and, since $(m - M) = 5\log(d/10)$, you would underestimate the distance to the galaxy. (And, since the Hubble Constant is V/D, you would therefore overestimate H_0.)

Malmquist Bias – An Example

Let's look at a specific example of Malmquist bias, where the assumed space density of objects is constant. The total number of objects within a distance, d, will then be

$$N(d) = \int_0^d \rho\, r^2 dr = \frac{1}{3}\rho\, d^3 \tag{16.09}$$

For stars of absolute magnitude M, d is related to apparent magnitude, m, via (16.08), so

$$N(m) = \frac{1}{3}\rho\, 10^{0.6(m-M)+3} = \frac{1}{3}\rho\, e^{1.382(m-M)+6.908} \tag{16.10}$$

In other words

$$N(m) \propto e^{1.382m} \tag{16.11}$$

But note that $N(m)$, the total number of objects brighter than m, is just the integral of the observed luminosity function down to apparent magnitude m,

$$N(m) = \int_{-\infty}^{m} A(m)dm \tag{16.12}$$

so

$$A(m) = \frac{dN(m)}{dm} \propto 1.382e^{1.382m} \tag{16.13}$$

and

$$\frac{d\ln A(m)}{dm} = 1.382 \tag{16.14}$$

This means that the objects in the sample will be brighter than the norm by a factor that goes as

$$\langle M \rangle = M_0 - 1.382\,\sigma^2 \tag{16.15}$$

So, in order to derive the correct absolute magnitude, you need to know the dispersion of the distribution *and* have some idea of the true space distribution of objects.

Malmquist Bias and the Determination of σ

The situation is even worse than it sounds. Let's use the same magnitude limited sample of objects, and try to estimate σ. To do this, we start with (16.05)

$$\frac{dA(m)}{dm} = \frac{M_0}{\sigma^2} A(m) - \frac{1}{\sigma^2} \int_0^\infty M \Phi(M) \rho(r)\, r^2 dr \tag{16.05}$$

and compute the second derivative

$$\begin{aligned} \frac{d^2 A(m)}{dm^2} &= \frac{M_0}{\sigma^2} \frac{dA(m)}{dm} - \frac{1}{\sigma^2} \int_0^\infty \frac{d}{dm} \left\{ M \Phi(M) \rho(r)\, r^2 \right\} dr \\ &= \frac{M_0}{\sigma^2} \left\{ \frac{M_0}{\sigma^2} A(m) - \frac{1}{\sigma^2} \int_0^\infty M \Phi(M) \rho(r)\, r^2 dr \right\} - \\ &\qquad \frac{1}{\sigma^2} A(m) + \frac{1}{\sigma^2} \int_0^\infty M \frac{(M - M_0)}{\sigma^2} \Phi(M) \rho(r)\, r^2 dr \end{aligned} \tag{16.16}$$

If we again divide by $A(m)$, we get

$$\frac{d^2 A(m)}{dm^2} \bigg/ A(m) = \frac{M_0^2}{\sigma^4} - \frac{\langle M \rangle M_0}{\sigma^4} - \frac{1}{\sigma^2} + \frac{\langle M^2 \rangle}{\sigma^4} - \frac{\langle M \rangle M_0}{\sigma^4} \tag{16.17}$$

Collecting terms gives

$$\sigma^4 \frac{d^2 \ln A(m)}{dm^2} = M_0^2 - 2 M_0 \langle M \rangle + \langle M^2 \rangle - \sigma^2 \tag{16.18}$$

or

$$\sigma^2 \left\{ 1 + \frac{d^2 \ln A(m)}{dm^2} \right\} = \left(M_0 - \langle M \rangle \right)^2 \tag{16.19}$$

But note: the right hand side of this equation is just the observed dispersion in magnitudes! It is not the same as the true dispersion.

$$\sigma^2_{\rm meas} = \sigma^2 \left\{ 1 + \sigma^2 \frac{d^2 \ln A}{dm^2} \right\} \tag{16.20}$$

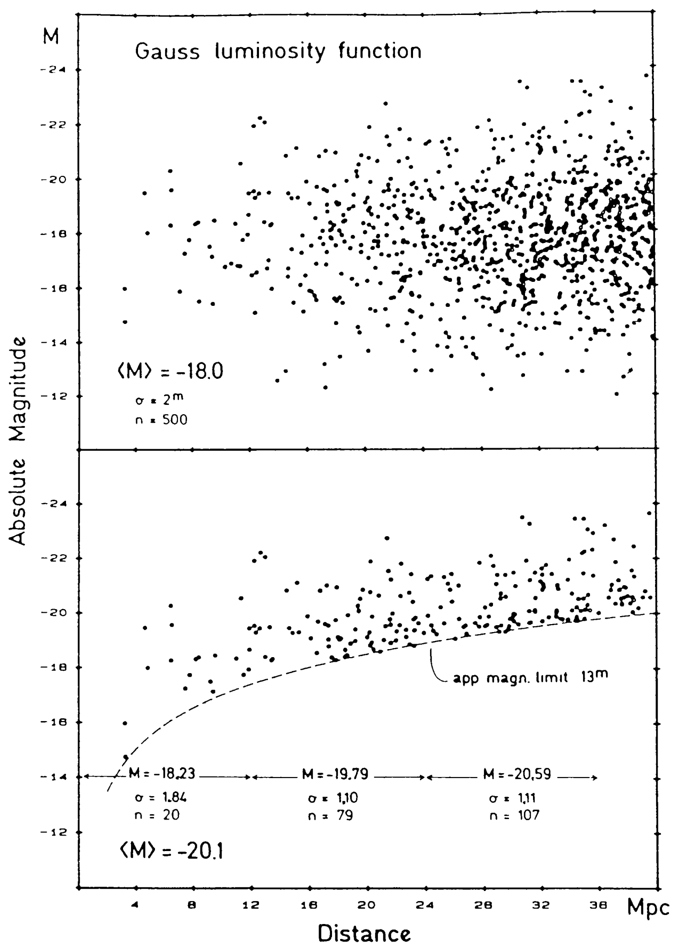


Fig.1. Upper panel: Monte Carlo distribution in distance and absolute magnitude of 500 galaxies within 38 Mpc. Constant space density and a mean absolute magnitude of $\langle M \rangle = -18^m$ with a Gauss standard deviation of $\sigma_M = 2^m$ are assumed. Lower pannel: The same sample cut by an apparent-magnitude limit of $m = 13^m.0$. Note the increase of the galaxian luminosities with increasing distance and the small effective (observable) scatter σ_M within individual distance intervals.

Malmquist Bias and the Tully-Fisher Relation

[Teerikorpi 1997, *A.R.A.A.,* **35,** 101]

The previous derivations for Malmquist bias apply if the intrinsic luminosity function of objects is a Gaussian with some dispersion. In the case of the Tully-Fisher relation, we are not working with a single mean magnitude, but with a relation between magnitude and rotation velocity. Under these circumstances, the mathematics of correcting for Malmquist bias is more complicated, but, in general, the result is the same. The effect of Malmquist bias is to change the slope of the Tully-Fisher relation by an amount that goes as the square of the dispersion about the mean line; the more the scatter, the steeper the apparent slope and the more you underestimate distance. As a result, Tully-Fisher arguments often focus on the amount of scatter in relation. For instance, one observes a set of galaxies in a cluster, and measures the slope and scatter in the Tully-Fisher relation. Is the scatter intrinsic to the Tully-Fisher relation, or is there depth and/or foreground-background contamination in the cluster? Depending on your point of view, you can use the observations to support a small or large Hubble Constant.

(The paragraph above may be debated by some. In particular, one argument goes that if you take a velocity-line width limited sample of galaxies, or select objects by H I line width (instead of optical magnitude), you can avoid the issue of Malmquist bias completely. But these formulations have their own statistical problems.)

In the 1990's, the debate about the Hubble Constant largely came down to an argument about the intrinsic scatter in the Tully-Fisher relation. In general, high Hubble Constant groups claimed that the scatter about the Tully-Fisher line was about $\sim$

0.3 mag; low Hubble-Constant people said the scatter is closer to ~ 0.7 mag. Since distance goes as the square of σ, this translates into a $\sim$ 0.6 mag difference in distance modulus, or almost a factor of 2!

The answer seems to be that the Tully-Fisher relation does indeed have a small amount of cosmic scatter. Observations in the Ursa Major Group show a dispersion of about the mean of $\sigma \sim 0.35$ mag. The larger ($\sigma \sim 0.8$ mag) dispersion in the Virgo Cluster seems to be due to the intrinsic depth of the cluster: Virgo is elongated along our line-of-sight. Note that the dispersion in the Tully-Fisher relation is small, and that the rotation rate of spirals is largely defined by the mass in their dark matter halos, the precision of the relation says a lot about the uniformity of the formation of disk systems.

Note also that the scatter in the relation depends on color: the B-band relation has significantly more scatter than the infrared law. This is most likely due to the effect of dust, and the deemphasizing of blue stars. (Observations in the blue record the young stars, but IR measurements probe the bulk of the stellar mass.)

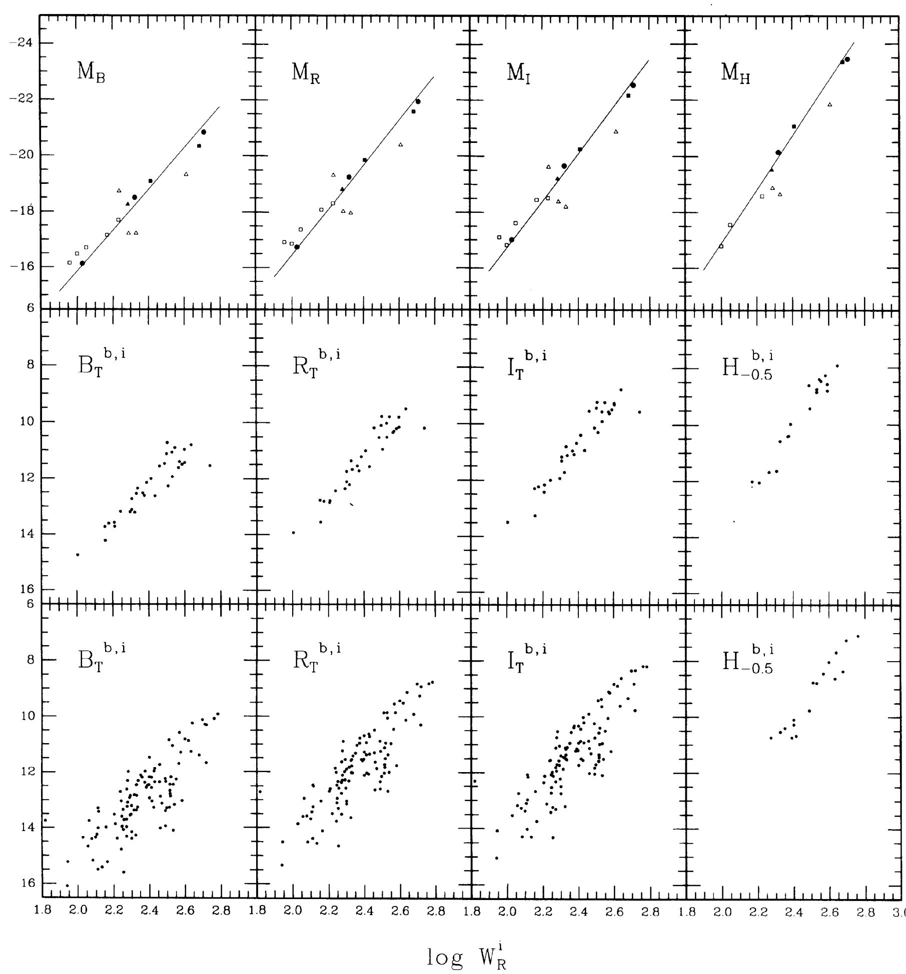


FIG. 11—*B*-, *R*-, *I*-, and *H*-band Tully–Fisher relations for the Local Calibrators (top), Ursa Major cluster members (middle), and Virgo cluster members (bottom). It is apparent from the figures that the slope of the relations increases going to longer wavelengths and the dispersion decreases. The variation in slope is thought to arise from the differing contributions to the observed bandpass made by greater fraction of young stars found in the lower-luminosity systems. The smaller dispersion at longer wavelengths is likely due to a reduction in the sensitivity to these effects, as well as those expected from extinction variations. Note the much larger dispersion found for the Virgo cluster data.